Formulation of ray origin and direction for panorama rendering

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Introduction

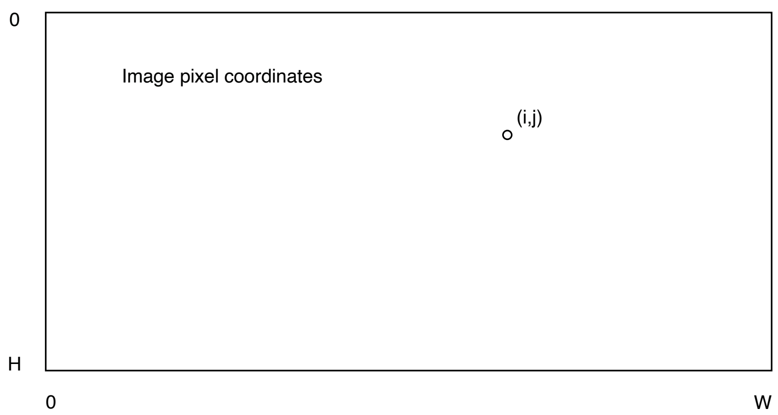
The following defines the origin and unit view direction for a pair of stereoscopic cameras associated with any position on an equirectangular or cylindrical panoramic projection. This is defined for zero parallax located at infinity or some fixed distance away. The former is typical for head mounted displays while the later is required for projection based stereoscopic viewing where zero parallax should be at the screen distance.

Coordinate conventions

The z axis is considered “up”, x to the “right”, y is “forward”. This is a right handed coordinate system.

Longitude and latitude

Mapping from equirectangular panoramas or cylindrical panoramas involves, as a first step, deriving the longitude and latitude of any point (i,j) in the image.



For an equirectangular panorama

$$longitude = \pi\left(\frac{2i}{W} - 1\right)$$

$$latitude = \frac{\pi}{2}\left(\frac{2(H-1-j)}{H} - 1\right)$$

For a cylindrical panorama

$$longitude = \pi\left(\frac{2i}{W} - 1\right)$$

$$latitude = latitude_{max}\left(\frac{2(H-1-j)}{H} - 1\right)$$

Where

$$latitude_{max} = 2atan\left(\frac{H\pi}{W}\right)$$

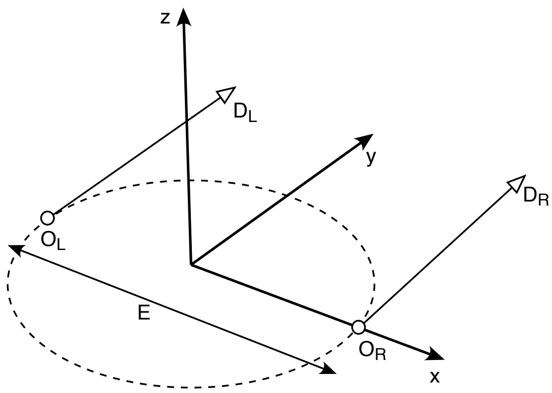
Noting that in the conventions used here

- For both equirectangular and cylindrical panoramas the longitude ranges from -π to π horizontally.
- For equirectangular panoramas the latitude ranges from -π/2 to π/2.
- For cylindrical panoramas the latitude ranges from -$latitude_{max}/2$ to $latitude_{max}/2$.
- Longitude=0 and latitude=0 occurs in the center of the panorama image.
- The equations above assume floating point division.

For a ray casting algorithm two quantities need to be derived, the origin of the ray (O) and the unit direction vector (D) of the ray. These will be denoted as O_L, O_R and D_L, D_R for the respective eyes. For a stereoscopic panorama the main parameter required is the eye separation (E). In the case of a zero parallax distance not at infinity the actual zero parallax distance (Z_o) is a second parameter. The matrix for rotation about the z axis (R_z) is defined as follows

$$R_z = \begin{pmatrix} \cos(longitude) & \sin(longitude) & 0 \\ -\sin(longitude) & \cos(longitude) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Case 1: Equirectangular or cylindrical panorama, zero parallax at infinity. The y axis corresponds to longitude and latitude of zero. Rotations are positive clockwise.

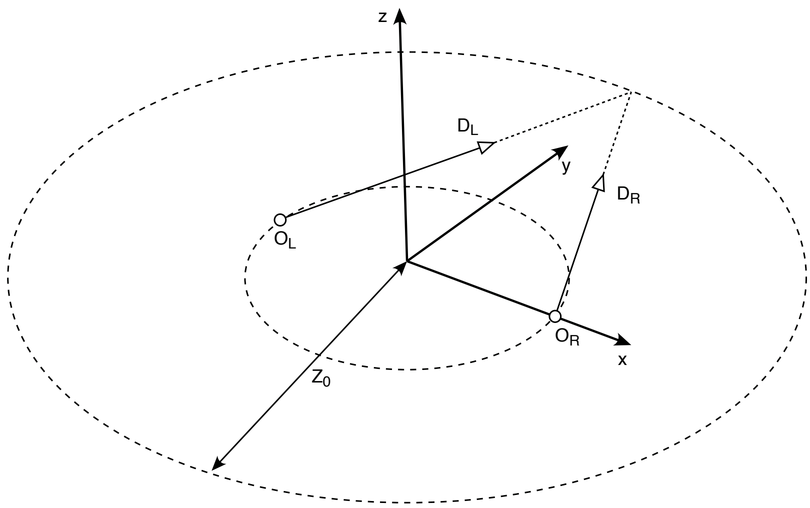


The above is intended to illustrate how the equations below are derived. The origin of the ray is just the positions on the (x,y) plane rotated by the longitude about the z axis. The ray direction is just the ray vector in the (y,z) plane also rotated by the longitude about the z axis.

$$O_L = R_z \begin{pmatrix} -E/2 \\ 0 \\ 0 \end{pmatrix} \qquad O_R = R_z \begin{pmatrix} E/2 \\ 0 \\ 0 \end{pmatrix}$$

$$D_L = D_r = R_z \begin{pmatrix} 0 \\ cos(latitude) \\ sin(latitude) \end{pmatrix}$$

Case 2: Equirectangular or cylindrical panorama, zero parallax at a fixed distance.



$$O_L = R_z\begin{pmatrix} -E/2 \\ 0 \\ 0 \end{pmatrix} \qquad O_R = R_z\begin{pmatrix} E/2 \\ 0 \\ 0 \end{pmatrix}$$

$$D_L = \left\| R_z\begin{pmatrix} 0 \\ Z_0 cos(latitude) \\ Z_0 sin(latitude) \end{pmatrix} - O_L \right\|$$

$$D_R = \left\| R_z\begin{pmatrix} 0 \\ Z_0 \cos(latitude) \\ Z_0 sin(latitude) \end{pmatrix} - O_R \right\|$$

Notes

- The unit direction vectors for case 2 (finite zero parallax distance) need to be normalised whereas they are formulated normalised for case 1 (zero parallax at infinity).
- It may seem counter intuitive that it doesn't matter whether one is dealing with a equirectangular or cylindrical panorama. This is because the formulation is in terms of longitude and latitude, noting the that the range of latitude values for the cylindrical panorama is -$latitude_{max}$ to $latitude_{max}$ rather than -π/2 to π/2.
- In practical terms, cylindrical panoramas with $latitude_{max}$ greater than 120 degrees become increasingly inefficient.
- If instead of defining the cylindrical panorama in terms of pixel height, it is more common to know the desired vertical field of view ($latitude_{max}$) and calculate what the panorama height should be. The expression is the same as that given earlier but solved for the height.

  $$H = \frac{W}{\pi}\tan\left(\frac{latitude_{max}}{2}\right)$$

  Obviously, the height of the equirectangular panorama should be just

  $$H = \frac{W}{2}$$