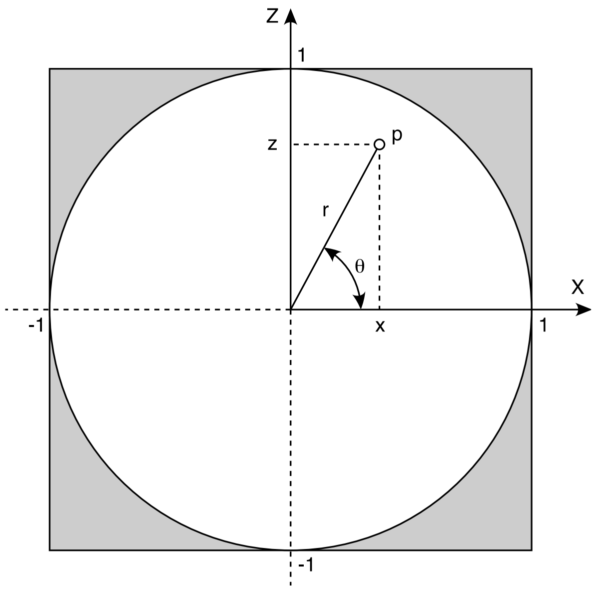
Further details on mapping fisheye projections into equirectangular images.

A fisheye projection is one of many ways of mapping points in 3D onto a plane. In the context of an image captured with a camera and fisheye lens, the key parameters are the camera position, orientation and the field of view of the fisheye lens. While a fisheye field of view is most commonly imagined to be 180 degrees, it is in fact defined for all angles and some physical fisheye lenses extend to 250 degrees. In the discussion here the fisheye projection will be assumed to be linear, that is, equal distances measured from the center of the image correspond to equal steps in the angle of the vector into the 3D scene (see later). This is typically referred to as an equal angle fisheye or true-theta fisheye. While real fisheye lenses may result in fisheye images that vary from this model, their images can be readily transformed so as to obey this relationship. This relationship between radius on the fisheye and 3D vector is key to any fisheye image remapping.
The conventions use for the fisheye image for deriving the mathematics are as shown in the following. The fisheye circle lies in the (x,z) plane, located at the origin and the image bounds normalised to lie between -1 and 1. The camera is imagined to be at the origin looking down the positive y axis.



Any 2D point p on a this fisheye image can be parameterised in polar coordinates (r,θ). These r, $\emptyset$, θ are given by

$$r = \sqrt{x^2 + z^2}$$

$$\theta = atan2(z, x)$$

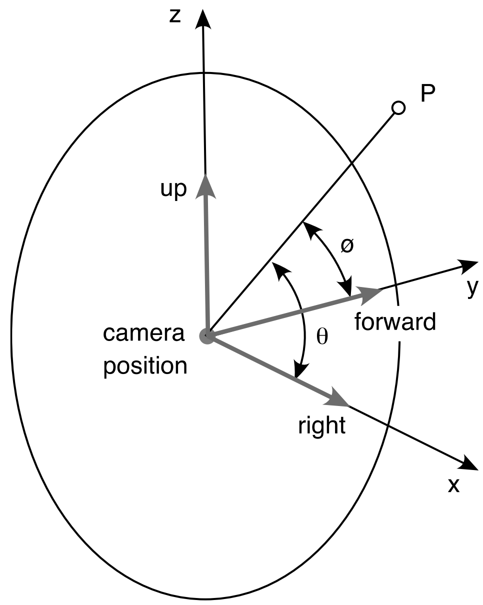
We define an additional angle $\emptyset$ as

$$\emptyset = \frac{\emptyset_{max} r}{2}$$

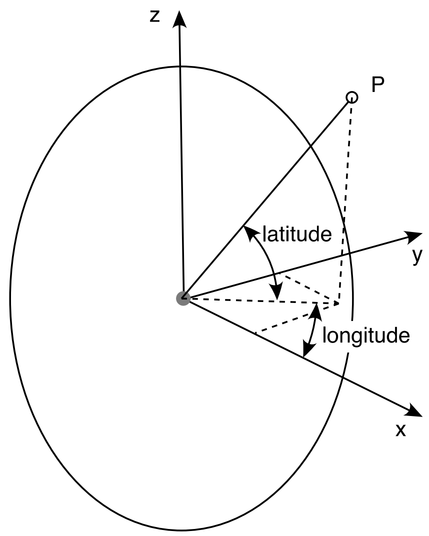
Where $\emptyset_{max}$ is the field of view of the fisheye lens. It is the linear relationship between radius r on the fisheye image and $\emptyset$ that is the key mapping for a fisheye projection.
The corresponding 3D vector P and the meaning of $\emptyset \; and \; \theta$ is illustrated in the following as well the camera conventions. Some special cases

- When r=0, this corresponds to the center of the fisheye image and $\emptyset = 0$ corresponding to a 3D vector along the optical axis of the fisheye lens.

- When r=1, this corresponds to positions around the rim of the fisheye circle. This corresponds to 3D vectors at angles of half the maximum field of view.
- Using the conventions presented here, θ is the same value in the 2D image and the 3D vector.



When mapping a fisheye image into an equirectangular projection one needs to calculate the longitude and latitude defined, conventions used here are as below

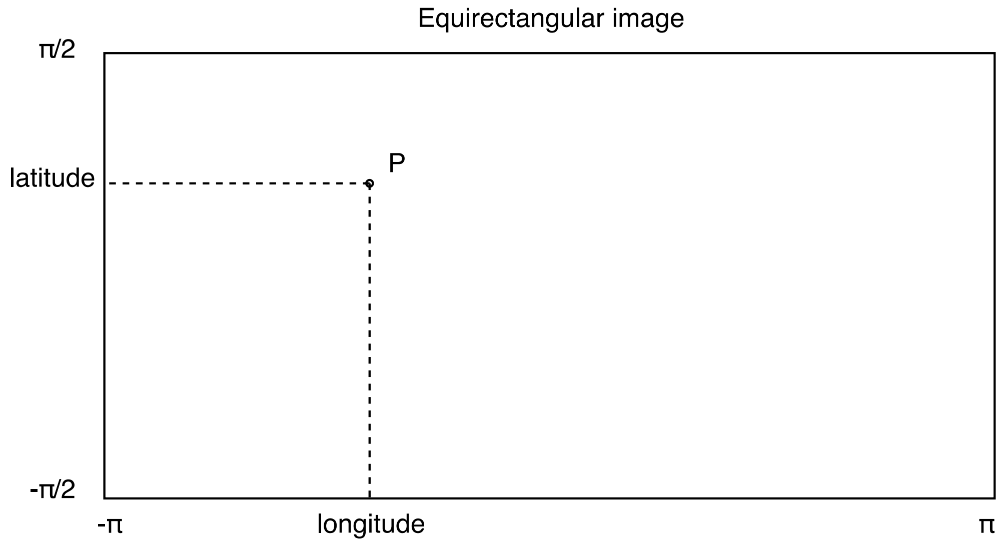


So

$$longitude = atan2(P_y, P_x)$$

$$latitude = atan2(P_{z,}, \sqrt{P_x + P_y})$$

Note that the exact definition of the longitude can vary depending on where on wants "0" longitude to be located. If 0 degrees longitude is to be located in the center horizontally of the equirectangular image then the longitude angle in the above should be between the y axis and the projection onto the x-y plane.


Equirectangular image
π/2
latitude
P
-π/2
-π
longitude
π